Tethered galaxies and the expanding space paradigm

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Abstract

I examine the counterintuitive claim from Davis et.al. [1] that a stationary or receding galaxy in an expanding universe may exhibit blueshift, even when the universe contains no gravitating matter. I show that with more precise calculations the effect disappears for the empty universe. I also discuss how this is related to the currently popular idea of 'expanding space', looking at how this idea arose, and why I think it has serious flaws.

1 Special and General relativity

In 1908 Hermann Minkowski claimed

> Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality

But it didn't happen. In cosmology today the favorite explanation of the expansion of the universe is the balloon model, where space is compared to the rubber surface of a balloon and the proper time of comoving objects is taken as *the* time coordinate for the universe. Far from fading into obscurity, space now has an almost tangible representation. So how did this happen? Well if you asked the question on an internet forum, then the answer would be straightforward. Minkowski was talking about special relativity, but the large scale behaviour of the universe is not governed

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by special but by general relativity, which supports the balloon model. However, on thinking about it this is no answer at all. Firstly, if space is like a rubber surface, then this will still hold on a local scale, but general relativity starts from the basis that spacetime is locally Minkowski spacetime. Secondly, on a large scale general relativity introduces general covariance. Far from separating out space and time, this seems to unite them more convincingly - general relativity is like special relativity only more so.

2 Eddington vs Milne

This paper is intended to be a look at the current view of the nature of space, rather than being a presentation of historical material. However, it is useful to look at how the idea of expanding space came about, and what objections there were to it.

Sir Arthur Eddington's 1919 solar eclipse expedition was instrumental in the acceptance of general relativity, and he was a strong supporter of the theory. He took up Einstein's idea of a universe which curved back on itself and so was finite. Eddington also believed in a positive cosmological constant, even after Hubble's work showed that this was not necessary. In particular Eddington put forward the balloon model of space, in which it wasn't so much that galaxies moved through space, it was more that the space in between them was expanding.

Edward A. Milne on the other hand didn't like the new ideas coming from general relativity. He thought that space was just space and that objects moved through it as they always had done. He claimed that assigning structure to space was a re-introduction of the ether[2, p.2]. Milne proposed his own version of cosmology, which was essentially a special relativistic viewpoint. This suggested that gravity cancelled out on cosmological scales, but Milne didn't see this as a flaw - after all Newton's argument for a static universe was based on a similar argument. Now it often happens in science that two seemingly opposite points of view are reconciled to be parts of the same theory. It was soon realised that the special relativistic cosmology could be seen as that predicted by general relativity for a zero density, zero cosmological constant universe. Hence one might expect that the two different viewpoints would eventually merge into one. After all, the only difference was one of choice of the coordinate systems, and swapping between different coordinate systems is all in a days work for a general relativist. However, it seems that no one wanted it to happen. Milne didn't want his ideas to be absorbed into general relativity and in his later work, such as his 1948 book [3] he gave greater emphasis to the differences. From the general relativists' point of view there were two problems. Firstly if Milne's ideas were taken seriously, then a universe without gravity on cosmological scales would have to be accepted as a possibility. Secondly they would show that most cosmological observations could be explained without needing general relativity, and when it was brought in it would be as a small modification to a special relativistic universe.

3 Current ideas on expanding space

There have been plenty of objections to the idea that expanding space is necessary to explain the expansion of the universe, e.g. [4, p. 739] ,

> To try to pinpoint where those cubic kilometers of space get born is a mistaken idea because it is a meaningless idea

and [5, p. 87]

> An inability to see that the expansion is locally just kinematical also lies at the root of perhaps the worst misconception about the big bang. Many semi-popular accounts of cosmology contain statements to the effect that 'space itself is swelling up' in causing the galaxies to separate'

This has also been addressed in [6] and more recently in [7] and [8]. So has 'a pruning of the existential extravagances of "curved space","expanding space"' [9] taken place? As pointed out in [10], the answer is no. The popular view is still that space expands, and it is claimed that any other way of looking at things is confused and misconceived [12] [11] [13]. [11] has a list of such misconceptions. But when one interprets the items in this list in terms of special relativistic coordinates, in which recession at the speed of light equates to infinite redshift, much of the confusion disappears. It's true that not all of it does, and that this choice of coordinates is seldom stated explicitly. However, to insist that coordinates must be interpreted as comoving coordinates, thus transferring the focus of interest from the cosmological event horizon to the hubble sphere, seems to me to be counterproductive.

4 The tethered galaxy in an empty universe

[1] In [1] Davis, Lineweaver and Webb examine the behaviour of a 'tethered galaxy', that is a hypothetical object at a constant distance from us, within a cosmological

[1] The results obtained in this and the following sections are similar to those obtained by Clavering [14], but I feel that my approach gives a clearer view of the problems with using comoving coordinates. See also [15]

context. According to special relativity such an object should not show any blueshift or redshift. The calculations in [1], however, show that such a galaxy would exhibit a blueshift, even in the case of the universe without gravitating matter or a cosmological constant - the (0,0) universe - which one would expect to be equivalent to the universe of special relativity. This gives strong support to the claim that the universe cannot be thought of in special relativistic terms. However, a more accurate calculation shows that the object cannot in fact be considered stationary with respect to us, and that there is no blueshift for a truly stationary object.

A homogeneous, isotropic universe is considered using the Friedman-Robertson-Walker metric, which can be given comoving coordinates (only radial distances are considered)

$$ds^2 = -dt^2 + a(t)^2 d\chi^2$$

Here $a(t)$ depends on the parameters of the model being considered, in particular on the density of gravitating matter and the cosmological constant. Here we consider the (0,0) case in which $a(t) = t/t_0$. (Note: all calculations use c=1)

The galaxy is tethered at a constant distance until proper time t_0 at which time its position is given by $\chi = \chi_0$. We take $\chi_0 > 0$, and the galaxy is taken to be within the Hubble sphere so that its motion does not exceed the local speed of light. (The coordinates are scaled so that $a(t_0) = 1$). In [1] constant distance is taken to mean constant $a(t)\chi$, so that the initial condition is given by $a_0\dot{\chi}_0 = -\dot{a}_0\chi_0$. Suppose, however, that we wanted to measure the distance to such an object. We would send a light signal and see how long it took to receive a return signal. Suppose we (at $\chi = 0$) send a light signal at time t_1 which meets the object at coordinate (χ, t) and it is reflected back to be received at time t_2. Then we have the formula for χ (see e.g. [16]) :

$$\chi = \int_{t_1}^{t} \frac{dt}{a(t)} = \int_{t}^{t_2} \frac{dt}{a(t)}$$

This means that $t_2 - t_1 = 2t \sinh \chi$ in the (0,0) universe. Assuming constant $t\chi$ we see that the light travel time tends to zero as $t \to \infty$ and so the object actually seems to be approaching us asymptotically. This has two problems. Firstly it does not represent constant distance. Secondly, the object is not supposed to be under the influence of any forces, but it appears to be decelerating.

5 The untethered galaxy

While the object is tethered it might be argued that choice of what one regards as constant distance is just a matter of preference. Clavering [14] has argued why radar distance is a good definition, but comoving distance ($a\chi$) also seems reasonable. The deciding point is what happens when the tether is removed, in particular in the (0,0) case. In the absence of forces one would expect the object to stay where it is. A calculation in [1] shows that (in the (0,0) case) $t\chi$ remains constant after the tether is released. Hence constant $a(t)\chi$ seems to be the correct definition of constant distance. This calculation, however, uses a non-relativistic approximation and so is not precisely correct. Although this would seem unimportant, it does in fact lead to the above non-physical result for large t . So the question is, what does the correct calculation predict for an object with the given initial conditions? It is possible to repeat the calculation in [1] including relativistic effects, but here we proceed via the geodesic equation: (Note that a dot indicates a derivative with respect to cosmological time t)

$$\ddot{\chi} = t\dot{\chi}^3/t_0^2 - 2\dot{\chi}/t$$

Clearly $t\chi$ constant is *not* a solution. The solutions are of the form

$$t\dot{\chi} = t_0 \tanh(w - \chi/t_0)$$

leading to

$$t \sinh(w - \chi/t_0) = constant$$

Taking the initial condition $t_0\dot{\chi}_0 = -\chi_0$, then as $t \to \infty$ we get

$$\chi \to wt_0 = \chi_0 - t_0 \tanh^{-1}(\chi_0/t_0) < 0$$

This initial condition does not represent a galaxy at constant distance. It represents a galaxy which starts at $\chi > 0$ and moves towards us, passing us and joining the Hubble flow on the other side of the sky. This explains why such a galaxy is seen to be blueshifted.

Note that if the tether is taken to represent constant radar distance then we have $t \sinh(\chi/t_0) = constant$ which is the solution of the geodesic equation with $w = 0$, and so the object will continue on the same geodesic when untethered. (w is simply the speed in 'special relativistic' coordinates) This shows that radar distance is the more reasonable definition of distance in this context.

6 The tethered galaxy in a non-empty universe

The above discussion indicates that 'radar distance' is strongly preferable when considering a tethered galaxy, and eliminates the blueshift in the empty universe. The question then arises, what happens in a non-empty universe? Consider the matter dominated, critical density universe given by $a = t^{\frac{2}{3}}$. Then the calculation of radar distance proceeds as follows:

$$\chi = \int_{t_1}^{t} u^{\frac{2}{3}} du = 3(t^{\frac{1}{3}} - t_1^{\frac{1}{3}})$$

So

$$t_1 = (t^{\frac{1}{3}} - \chi/3)^3 = t - t^{\frac{2}{3}}\chi + t^{\frac{1}{3}}\chi^2/3 - \chi^3/27$$

Likewise

$$t_2 = (t^{\frac{1}{3}} + \chi/3)^3 = t + t^{\frac{2}{3}}\chi + t^{\frac{1}{3}}\chi^2/3 + \chi^3/27$$

Hence radar time and distance are given by

$$T_r = t + t^{\frac{1}{3}}\chi^2/3, X_r = t^{\frac{2}{3}}\chi + \chi^3/27$$

Now consider the case when the object has constant radar distance, and emits a signal from point (χ, t), which is received by us at time T.

$$\chi = \int_{t}^{T} u^{\frac{2}{3}} du = 3(T^{\frac{1}{3}} - t^{\frac{1}{3}})$$

$$T = (t^{\frac{1}{3}} + \chi/3)^3 = t + t^{\frac{2}{3}}\chi + t^{\frac{1}{3}}\chi^2/3 + \chi^3/27$$

For constant X_r we have

$$0 = (\tfrac{2}{3}t^{-\frac{1}{3}}\chi + \dot{\chi}(t^{\frac{2}{3}} + \chi^2/9))$$

Let $\beta = t^{-\frac{1}{3}}\chi^2/9$. Then:

$$\dot{\chi} = -\tfrac{2}{3}t^{-1}\chi(1+\beta)$$

$$\dot{T} = 1 + \tfrac{1}{3}t^{-\frac{2}{3}}\chi^2/3 + \tfrac{2}{3}t^{\frac{1}{3}}\chi\dot{\chi} = 1 + \beta - \tfrac{4}{9}t - \tfrac{2}{3}\chi^2/(1+\beta)$$
$$= 1 + \beta - 4\beta/(1+\beta) = (1-\beta)^2/(1+\beta)$$

Note that t is cosmological time. The object is moving with respect to the comoving background, and so its proper time τ will differ from this, but can be obtained directly from the metric $d\tau^2 = dt^2 - a(t)^2 d\chi^2$ giving.

$$\dot{\tau}^2 = 1 - t^{\frac{4}{3}}\dot{\chi}^2 = 1 - \tfrac{4}{9}t - \tfrac{2}{3}\chi^2/(1+\beta)^2$$

$$= 1 - 4\beta/(1+\beta)^2 = (1-\beta)^2/(1+\beta)^2$$

$$\dot{\tau} = (1-\beta)/(1+\beta)$$

Considering the frequency of the emitted signal, we see that the redshift is the rate of change of the time of observation with respect to the proper time of emission. Hence

$$z + 1 = \dot{T}/\dot{\tau} = 1 - \beta < 1 \text{ so } z < 0$$

This means that the object is blueshifted, agreeing with the findings in [1] for comoving distance.

This isn't necessarily counterintuitive. In the matter dominated universe, each comoving observer may think of themselves at the bottom of a gravitational potential well, and so would expect a signal from an object at constant distance to be blueshifted.

Similar calculations can be done for other metrics:

Description of Universe	**Metric**	**Redshift of tethered galaxy?**	**Notes**
Empty	$a = t$	Zero, (as indicated above).	
Radiation dominated, critical density	$a = t^{\frac{1}{2}}$	Blueshift (as with matter).	Comoving distance and radar distance coincide.
Cosmological constant dominated	$a = e^{Ht}$	Redshift	Radar distance is a function of comoving distance, so the definitions of constant distance coincide.

7 Discussion

General relativity allows considerable freedom in choosing coordinates to represent the metric of a given spacetime. In cosmology the proper time of comoving objects is usually taken as the time coordinate, giving coordinates for the (0,0) case of $ds^2 = -dt^2 + (t/t_0)^2 d\chi^2$. Nevertheless there is the 'Special relativistic' alternative $ds^2 = -dt^2 + dx^2$ [6],[17]. Note that these represent the same metric and so should always lead to the same result in any calculation.

In comoving coordinates there are objects moving apart faster than light. This is usually explained by saying that it is the expansion of space between the objects rather than the motion of the objects themselves. Unfortunately this has lead to many misunderstandings. There is a belief that if an object is moving away from us faster than light then we cannot see it - it is beyond a horizon. Also it is implied that space has physical properties, and so it is assumed that it influences the matter within it, causing it to join in with the Hubble flow. The belief that expanding space is needed to explain why the galaxies are receding may then encourage the pre-Newtonian idea that a moving object requires a cause to stay in motion.

Adding to the confusion is the the recent discovery that the universe may have a positive cosmological constant due to vacuum energy, is which case it is more reasonable to think of space as having physical properties and objects merging into the Hubble flow. In academic circles it is always clear whether the model which is being discussed involves a cosmological constant or not. Unfortunately in popular accounts this is often not the case, and the distinction between different models is not made. This is a particular problem in discussions of why local objects do not join in with the expansion.

A large part of this confusion has been addressed by Lineweaver and Davis in [11]. They argue that thinking in 'special relativistic' terms is part of the source of the confusion. I would argue that it is the other way round, and that it is the insistence on sticking to comoving coordinates which has lead to the attribution of physical causes for what is actually an effect of coordinate choice [18]. In this paper I have shown the problems with one argument against special relativistic intuition, that a stationary object in an empty universe can be blueshifted. I hope that the reader is encouraged to a greater belief in such intuition.

8 Some comments about Milne's cosmology

Milne [19] used as a starting point the idea that the homogeneity of the universe was so fundamental that it might be used as a replacement for general relativity. e.g. from [20]

> The procedure is very different from that of Milne, Zeits f. Astrophys 6, 1 (1933) who would regard the homogeneity of the universe as a fundamental principle from which even the laws of gravitation might be deduced.

But one finds the claim that the most prominent feature of Milne's cosmology is its inhomogeneity, for instance [21]

> Rejection of the expanding space paradigm in favor of Milne's picture of the expansion in fixed Euclidean space contradicts general relativity and leads to the conclusion that the universe possesses a centre and an edge

and [22]

> the Milky Way sitting at the middle of a fixed-metric with a universal explosion of galaxies in all directions (as seen in, for example, an early model proposed by Milne).

It is interesting to see how, following the fall from favour of Milne's cosmological views, reports of his contributions can distort them beyond recognition.

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